



General Certificate of Education  
Advanced Subsidiary Examination  
January 2010

## Mathematics

## MPC1

### Unit Pure Core 1

Monday 11 January 2010 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 The polynomial  $p(x)$  is given by  $p(x) = x^3 - 13x - 12$ .

(a) Use the Factor Theorem to show that  $x + 3$  is a factor of  $p(x)$ . (2 marks)

(b) Express  $p(x)$  as the product of three linear factors. (3 marks)

2 The triangle  $ABC$  has vertices  $A(1, 3)$ ,  $B(3, 7)$  and  $C(-1, 9)$ .

(a) (i) Find the gradient of  $AB$ . (2 marks)

(ii) Hence show that angle  $ABC$  is a right angle. (2 marks)

(b) (i) Find the coordinates of  $M$ , the mid-point of  $AC$ . (2 marks)

(ii) Show that the lengths of  $AB$  and  $BC$  are equal. (3 marks)

(iii) Hence find an equation of the line of symmetry of the triangle  $ABC$ . (3 marks)

3 The depth of water,  $y$  metres, in a tank after time  $t$  hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \quad 0 \leq t \leq 4$$

(a) Find:

(i)  $\frac{dy}{dt}$ ; (3 marks)

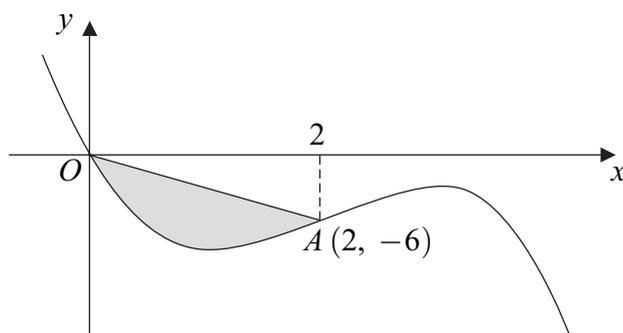
(ii)  $\frac{d^2y}{dt^2}$ . (2 marks)

(b) Verify that  $y$  has a stationary value when  $t = 2$  and determine whether it is a maximum value or a minimum value. (4 marks)

(c) (i) Find the rate of change of the depth of water, in metres per hour, when  $t = 1$ . (2 marks)

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when  $t = 1$ . (1 mark)

- 4 (a) Show that  $\frac{\sqrt{50} + \sqrt{18}}{\sqrt{8}}$  is an integer and find its value. (3 marks)
- (b) Express  $\frac{2\sqrt{7} - 1}{2\sqrt{7} + 5}$  in the form  $m + n\sqrt{7}$ , where  $m$  and  $n$  are integers. (4 marks)
- 5 (a) Express  $(x - 5)(x - 3) + 2$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are integers. (3 marks)
- (b) (i) Sketch the graph of  $y = (x - 5)(x - 3) + 2$ , stating the coordinates of the minimum point and the point where the graph crosses the  $y$ -axis. (3 marks)
- (ii) Write down an equation of the tangent to the graph of  $y = (x - 5)(x - 3) + 2$  at its vertex. (2 marks)
- (c) Describe the geometrical transformation that maps the graph of  $y = x^2$  onto the graph of  $y = (x - 5)(x - 3) + 2$ . (3 marks)
- 6 The curve with equation  $y = 12x^2 - 19x - 2x^3$  is sketched below.



The curve crosses the  $x$ -axis at the origin  $O$ , and the point  $A(2, -6)$  lies on the curve.

- (a) (i) Find the gradient of the curve with equation  $y = 12x^2 - 19x - 2x^3$  at the point  $A$ . (4 marks)
- (ii) Hence find the equation of the normal to the curve at the point  $A$ , giving your answer in the form  $x + py + q = 0$ , where  $p$  and  $q$  are integers. (3 marks)
- (b) (i) Find the value of  $\int_0^2 (12x^2 - 19x - 2x^3) dx$ . (5 marks)
- (ii) Hence determine the area of the shaded region bounded by the curve and the line  $OA$ . (3 marks)

**Turn over for the next question**

**Turn over ►**

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7 A circle with centre  $C$  has equation  $x^2 + y^2 - 4x + 12y + 15 = 0$ .

(a) Find:

(i) the coordinates of  $C$ ; *(2 marks)*

(ii) the radius of the circle. *(2 marks)*

(b) Explain why the circle lies entirely below the  $x$ -axis. *(2 marks)*

(c) The point  $P$  with coordinates  $(5, k)$  lies outside the circle.

(i) Show that  $PC^2 = k^2 + 12k + 45$ . *(2 marks)*

(ii) Hence show that  $k^2 + 12k + 20 > 0$ . *(1 mark)*

(iii) Find the possible values of  $k$ . *(4 marks)*

**END OF QUESTIONS**